Truth tables

In propositional logic, we use truth tables to identify whether an argument is valid or invalid.

The simplest, and most abstract logic we can study is called propositional logic. A proposition is a statement that can be either true or false; it must be one or the other, and it cannot be both.

The Connectives

There is a number of connectives which will allow us to build up complex propositions.

complex proposition: is a proposition made up of atomic propositions.

An atomic proposition is one whose truth or falsity does not depend on the truth or falsity of any other proposition.

The connectives we introduce are:

1- Conjunction (. / ^)
2- Disjunction (V)
3- Condition(→)
4- Biconditional (↔)
5- Negation (-)

1- Conjunction (And .)

Definition:

• Definition

If p and q are statement variables, the **conjunction** of p and q is "p and q," denoted $p \wedge q$. It is true when, and only when, both p and q are true. If either p or q is false, or if both are false, $p \wedge q$ is false.

summarized in a truth table.

Truth Table for (p ^q)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

and will be true iff both p and q are true.
The idea of a truth table for some formula is that it describes the behavior of a formula under all possible interpretations of the primitive propositions that are included in the formula.

2- Disjunction (Or V)

Any two propositions can be combined by the word ‘or’ to form a third proposition called the disjunction of the originals.

• Definition

If p and q are statement variables, the **disjunction** of p and q is “p or q,” denoted $p \vee q$. It is true when either p is true, or q is true, or both p and q are true; it is false only when both p and q are false.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

and will be true iff either p is true, or q is true, or both p and q are true.

3- Conditional (If. . . Then →)

• Definition

If p and q are statement variables, the **conditional** of q by p is “If p then q” or “p implies q” and is denoted $p \to q$. It is false when p is true and q is false; otherwise it is true. We call p the **hypothesis** (or **antecedent**) of the conditional and q the **conclusion** (or **consequent**).

Truth Table for $p \rightarrow q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

4- Biconditional (iff ↔)

• Definition

Given statement variables p and q, the **biconditional of p and q** is "p if, and only if, q" and is denoted $p \leftrightarrow q$. It is true if both p and q have the same truth values and is false if p and q have opposite truth values. The words *if and only if* are sometimes abbreviated **iff.**

Truth Table for $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

.

5- Negation (Not -)

• Definition

If p is a statement variable, the **negation** of p is "not p" or "It is not the case that p" and is denoted $\sim p$. It has opposite truth value from p: if p is true, $\sim p$ is false; if p is false, $\sim p$ is true.

Negation in truth table:

p	$\sim p$
T	F
F	T

Testing validity of argument form

To identify whether an argument is valid or invalid you need to:

1. Identify premises and conclusion of argument form.

2. Construct a truth table showing truth values of all premises and conclusion, under all possible truth values for variables.

3. A row of the truth table in which all the premises are true is called a **critical row.**

If there is a critical row in which conclusion is false, then it is possible for an argument of the given form to have true premises and a false conclusion, and so the argument form is invalid.

If conclusion in *every* critical row is true, then argument form is valid.

Example1:

Use truth table to Decide whether the following arguments are valid or invalid:

1-

If New York is a big city, then New York has tall buildings.

New York has tall buildings

Therefore, New York is a big city.

- The first step is to symbolize the argument as the following:

$(A \rightarrow B)$

B

―――

A

- There is only 2 perimeters in the formula , therefore there will be 4 rows in the table.

A	B	(A→B)	B	A
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

There is true premises and false conclusion in the third raw which means that the argument is invalid.

2-

If there is no government, then people will constantly be afraid of everyone else.
If people are constantly afraid of everyone else, then there will be no industry. Thus, if there is no government, then there will be no industry or business.

(-A→ B)
(B→ -C)

____
(-A→-C)

A	B	C	-A	-C	(-A→ B)	B→ -C)	(-A→-C)
T	T	T	F	F	T	F	T
T	T	F	F	T	T	T	T
T	F	T	F	F	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	F	T	F	F
F	T	F	T	T	T	T	T
F	F	T	T	F	F	T	F
F	F	F	T	T	F	T	T

We can see that there is no TTF in the table, which means that the argument is valid.