Libyan International University
Faculty of Business Administration

The Queuing Theory

Subject: Operation Research
instructor: Dr. Sabri Jobran
Topic: Queuing Theory

Done by :
Abdulrahman Al-tarhouni 1818

Table of Content

What is queuing theory?

Waiting for service is part of daily life. We wait for service in restaurants, we queue up to board a plane, and we line up for service in post offices. And the waiting phenomenon is not an experience limited to human beings: Jobs wait to be processed on a machine, planes circle in stack before given permission to land, and cars stop at traffic lights. Eliminating waiting altogether is not a feasible option because the cost of installing and operating the service facility can be prohibitive.

Queuing problems arises because either:

There is too less demand on the facilities .

There is too much demand on the facilities.

The importance of Queuing Theory

Holding up in line may be a portion of standard of living since as a prepare it has a few vital capacities, Lines are a reasonable and fundamental way of managing with the stream of clients when there are constrained assets. Negative results emerge in case a queue process isn't set up to bargain with overcapacity.

For case, when as well numerous guests explore to an online, site the site will moderate and crash in the event that it doesn't have a way to alter the speed at which it forms demands or a way to line guests.

Basic Elements of Queuing System

Total number of customers

It’s the number of people who want to get served.

The arrival of people

It’s the way of flock to place that provide service.

Formation of the queue for getting a service

The number of individuals that are taken into thought when they are standing in a line holding up to be served is known as shaping of a line, This handle of holding up to be served is known as lining up.

➢Characteristics of Queuing Process

Characterizing a Lining System Queuing models analyze how clients (counting individuals, objects, and data) get a benefit. A lining framework contains:

Arrival process. The entry handle is essentially how clients arrive, They may come into a line alone or in bunches, and they may arrive at certain interim or randomly.

Behavior. How do clients carry on when they are in line? A few could be willing to hold up for their put within the line; others may ended up restless and take off. However others might choose to rejoin the line afterward, such as when they are put on hold with client benefit and choose to call back in trusts of getting speedier benefit.

How clients are overhauled. This incorporates the length of time a client is overhauled, the number of servers accessible to assist the clients, whether clients are served one by one or in clusters, and the arrange in which clients are overhauled, moreover called benefit discipline.

. **Service discipline** Alludes to the run the show by which the following client is chosen. In spite of the fact that numerous retail scenarios utilize the “first come, to begin with served” run the show, other circumstances may call for other sorts of benefit. For case, clients may be served in arrange of need.

Queuing Disciplines:

- First come, first served (FCFS)
- Last come, first served (LCFS)
- Random selection for service (RSS)
- Priority queue.
- Preemptive / non preemptive.

• System Capacity.

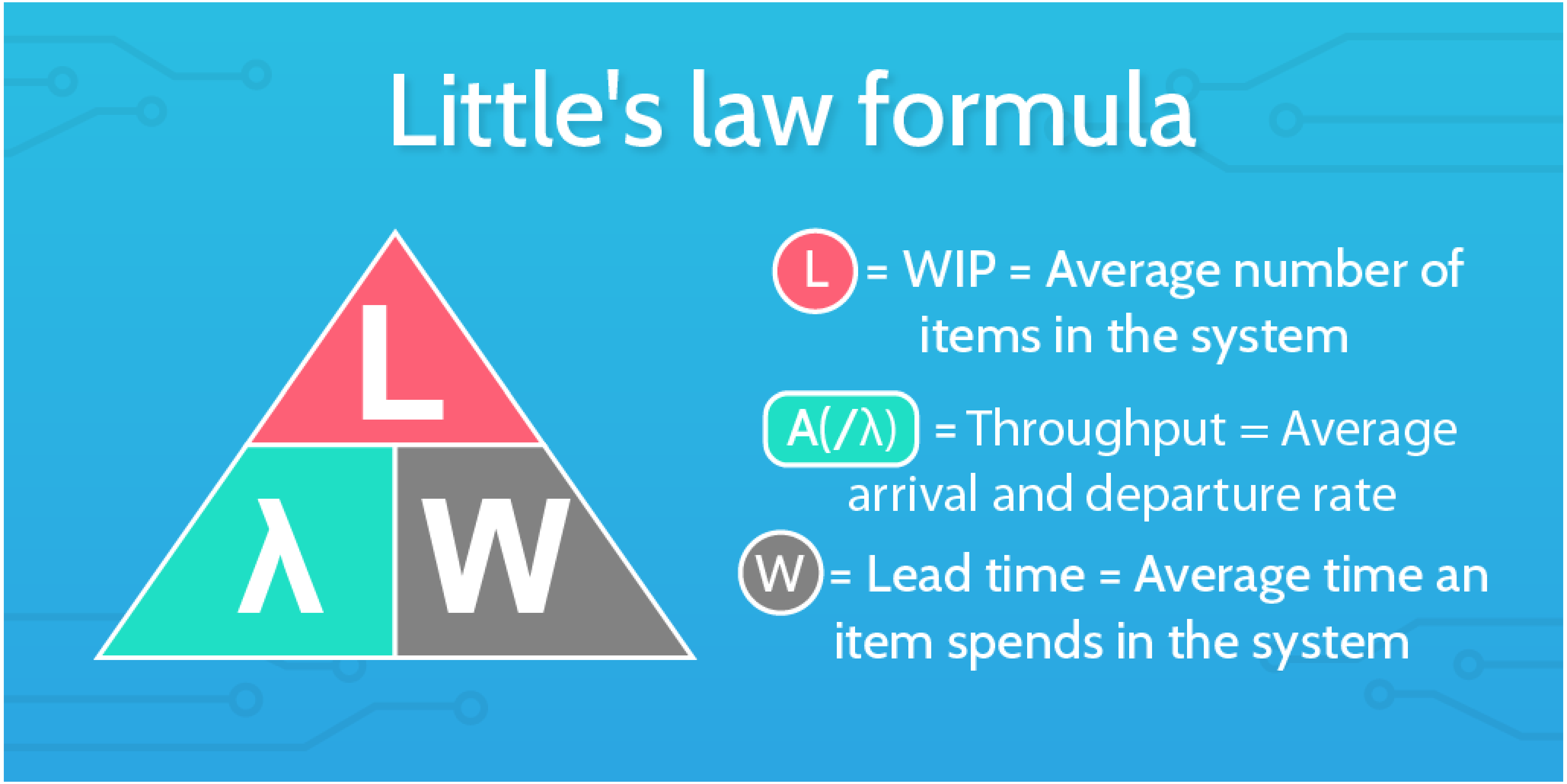
_ Finite / Infinite waiting room

➢What is the function of queuing theory

His job is to calculate the average of both the average wait and the average number of people waiting, and how long it takes to serve customer.

The little's Law

It is the waiting rate calculation



The little’s Law example

There is a restaurant where you are waiting for them to serve you, but you want to know how much time you will wait for them.

Suppose there are 21 people in the queue and the restaurant is served with 3 people per minute, how is this calculated?

$$\text{Throughput} = \frac{\text{WIP}}{\text{Lead Time}}$$

$$\text{Lead Time} = \frac{\text{WIP}}{\text{Throughput}}$$

$$\frac{21 \text{ people in line}}{3 \text{ people served per minute}} = 7 \text{ minutes of waiting}$$

Kendall Notation

SYMBOLS

λ = mean arrival rate

μ = mean service rate per busy server

ρ = λ / μ = utilization factor

Source: Taha, Hamdy A. *Operations research: an introduction*. Vol. 790. Pearson/Prentice Hall, 2011.

SYMBOLS

n = number of units in the system

$P_n(t)$ = probability of exactly n customers in the system at time t

P_n = probability of exactly n customers in the system

SYMBOLS

W_s = expected waiting time per customer in the system

W_q = expected waiting time per customer in the queue

L_s = expected number of customers in the system

L_q = expected number of customers in the queue

Source: Taha, Hamdy A. *Operations research: an introduction*. Vol. 790. Pearson/Prentice Hall, 2011

Probability	$P = \frac{\lambda}{\mu}$
	$P_0 = \left(1 - \frac{\lambda}{\mu}\right)$
	$P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right)$
Unit rate	$L_s = \frac{\lambda}{\mu - \lambda}$
	$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = L_s - \frac{\lambda}{\mu}$
Time rate	$W_s = \frac{1}{\mu - \lambda} = \frac{L_s}{\lambda}$
	$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{L_q}{\lambda} = W_s - \frac{1}{\lambda}$

Exercise

If the berth of a port accommodates only one ship for shipping and the port official has noticed many times five ships waiting for unloading or loading, so the Administrator thought about putting in place alternatives to reduce the waiting situation by flying loading and unloading operations.

If the rate of arrival of ships is 21 ships per week and assuming that the workers of loading or unloading can service 4 ships per day and the number of daily working hours is 10 hours.

21 Arrival of ships = lambda

4 Serves rate = mu

Lambda =3
mu=4

Lambda used to be the same(days- or week) which mean 21/7=3 to equal the mu.

Exercise

Find the following:

1. Average time the server is busy
2. It is possible that there is no unit in the system
3. Probability of having five ships in the system
4. Average number of ships in the system
5. The rate of ships in line
6. Average time a ship takes to be in the system
7. The average time a ship takes to queue

Exercise

Average time the server is busy

$$P = \frac{\lambda}{\mu} = \frac{3}{4} = 0.75$$ is the average time of busy server

It is possible that there is no unit in the system

$$P_0 = \left(1 - \frac{\lambda}{\mu}\right) = \left(1 - \frac{3}{4}\right) = 0.25$$

The possibility if there s no unit in the system is 0.25

$P = \frac{\lambda}{\mu}$
$P_0 = \left(1 - \frac{\lambda}{\mu}\right)$
$P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right)$
$L_s = \frac{\lambda}{\mu - \lambda}$
$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = L_s - \frac{\lambda}{\mu}$
$W_s = \frac{1}{\mu - \lambda} = \frac{L_s}{\lambda}$
$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{L_q}{\lambda} = W_s - \frac{1}{\lambda}$

1. Probability of having five ships in the system

$$P_5 = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right) = \left(\frac{3}{4}\right)^5 \left(1 - \frac{3}{4}\right) = 0.0593$$

Is the probability of having 5 ships in the system

2. Average number of ships in the system

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{3}{4 - 3} = 3$$

Is the average number of ships in the system

3. The rate of ships in line

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = L_s - \frac{\lambda}{\mu} = 3 - 0.75 = 2.25$$

The rate of ships in line is 2.25

4. Average time a ship takes to be in the system

$$W_s = \frac{1}{\mu - \lambda} = \frac{L_s}{\lambda} = \frac{3}{3} = 1$$

Is the average time that ships take to be in the system

5. The average time a ship takes to queue

$$W_q = \frac{L_q}{\lambda} = \frac{2.25}{3} = 0.75$$

Is the average time a ship take to queue

Exercise2

Customers flock to a bank at a rate of 10 customers per hour. One teller works in the bank to serve customers at a rate of 4 minutes per customer, assuming that the customers are dispatched that the service time sells an exponential distribution and that the bank accommodates any number of customers, find the following.

Percentage of time the teller spends unemployed?

What is the average number of people waiting in line to receive a service ?

If I entered a branch at exactly the hour 9:15, when do you expect to leave it after obtaining the service?

What is the average number of customers in a branch?

What is the average customer's time spent waiting?

the resolution

Percentage of time the teller spends unemployed?

$$P_0 = \left(1 - \frac{\lambda}{\mu}\right) = \left(1 - \frac{10}{15}\right) = 0.333$$ **The time that teller spend unemployed**

What is the average number of people waiting in line to receive a service

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{10^2}{15(15-10)} = 1.33$$ **The average time of people waiting in line**

If I entered a branch at exactly the hour 9:15, when do you expect to leave it after obtaining the service?

$$W_s = \frac{1}{\mu-\lambda} = \frac{1}{15-10} * 60 = 12$$ which mean 12+9:15=9:27 **The expected time to get served is 9:27 min**

What is the average number of customers in a branch?

$$L_s = \frac{\lambda}{\mu-\lambda} = \frac{10}{15-10} = 2$$ **Is the average number of customer in branch**

What is the average customer's time spent waiting

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{10}{15(15-10)} = 0.1333 * 60 = 8$$ **The time that customer spent in waiting is 8 min**

Exercise

If there is a café, customers arrive at a rate of 7 customers per hour and one worker works in this café and this worker serves customers at a rate of 2 minutes. If the café accommodates any number of customers, find the following.

What is the average number of workers waiting in line to receive the service?

What is the average time a customer spends waiting?

Percentage of time a worker spends unemployed

Exercise

If there is a bank that provides exchange services and other services The time to serve the customer took 6 minutes, and the time you waited in line was 10 minutes. How many people were waiting?

Conclusion

The queue theory is important for organizing the customer's time to receive the service, and it is very useful for companies to give distinctive ideas, and recognition, which is one of the reasons that make you keep your customer, and also makes you creative and fast.

References

Jouini, O., & Roubos, A. (2013, March 20). On multiple priority multi-server queues with impatience - journal of the Operational Research Society. SpringerLink. Retrieved December 24, 2021.

Little's Law. Kanban Zone. (2020, July 1). Retrieved November 3, 2021.

Pirmoradi, Z., Griffey, P., Joshi, A., Raj, P., & Jones, D. (2017, November 20). *Little's Law: How to Analyze Your Processes (with stealth bombers): Process street: Checklist, Workflow and SOP software*. Process Street. Retrieved December 24, 2021.

Queue-it. (2021, August 10). *Queuing theory: Definition, history & real-life applications*. Queue. Retrieved November 8, 2021, from https://queue-it.com/blog/queuing-theory/.

Taha, Hamdy A. *Operations Research : An Introduction*. Upper Saddle River, N.J., Pearson/Prentice Hall, 2011

Team, T. I. (2021, December 7). Queuing theory and the science of waiting in line. Investopedia. Retrieved December 24, 2021.


Thank
You.